Spontaneous Decoherence in Coupled Quantum Kicked Rotators

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Abstract

Quantum mechanical behavior of coupled N-kicked rotators is studied. In the large N limit each rotator evolves under influence of the mean-field generated by surrounding rotators. It is found that the system spontaneously generates classical chaos in the large N limit when the system parameter exceeds a critical value. Numerical simulation of a quantum rotator coupled to a classical rotator supports this idea.

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1 Introduction

In spite of numerous efforts to define quantum chaos, there seems to exist many counter examples of genuine chaos in quantum systems. Among those the quasi-periodic time evolution of isolated bound systems and the limited diffusion due to the momentum localization in the periodically kicked quantum rotator are typical examples [1, 2, 3]. Although they are rather persuasive to give up searching quantum chaos, we still reserve a good reason to believe the seeds of chaos should exist in quantum system which grow to classical chaos. One way to observe it is in the time evolution of the Wigner function;

$$\frac{\partial f}{\partial t} = L[f]. \tag{1}$$

When the quantum Liouville operator L is expanded in terms of the Planck constant as

$$L = L_0 + \hbar^2 L_1 + \cdots, \tag{2}$$

the lowest order term generates the classical evolution,

$$L_0[f] = \sum_i \{H, f\}^{(i)}, \tag{3}$$

where $\{H, f\}^{(i)}$ represents the Poisson bracket with respect to the i-th canonical variables (q_i, p_i) of a system with the Hamiltonian $H(\{q_i\}, \{p_i\})$. Therefore, it is natural to search the origin of chaos in the region of $\hbar \sim 0$, *i.e.* the semi-classical limit. In fact, recent intensive investigations [4, 5] have revealed rich structures in this limit.

There is another way to observe the emergence of chaos. If it exists in the system which involves many degrees of freedom. Since the most characteristic property of a macroscopic system is the large number of degrees of freedom involved in describing the motion, it is natural to consider a system with infinite number of degrees of freedom as the realistic classical limit. In this case we expect that quantum mechanical phases of each degree of freedom interfere mutually, and are wiped out in chaotic environment resulting the classical behavior of the Wigner function, *i.e.* the classical distribution function [6].

When the number of degrees of freedom N involved in a system tends to infinity, the mean-field approximation often becomes exact due to the large number theorem. In such a case the Wigner function can be approximated in a product form [7],

$$f(\{q_i\}, \{p_i\}; t) \approx \prod_{i=1}^{N} f_i(q_i, p_i; t), \tag{4}$$

of functions corresponding to the N sub-systems. The Wigner function for the i-th sub-system then evolves as

$$\frac{\partial f_i}{\partial t} = L_i^{(t)}[f_i], \tag{5}$$

with the appropriately defined mean-field operator, $L_i^{(t)}$. In general, this operator depends on the temporal distributions of surrounding sub-systems. Thus the mean-field Liouville equation (5) becomes non-linear and time-dependent. It is our expectation that the non-linearity and the time-dependence induce chaos, and chaos destroys quantum phases.

In the next section we shall describe time evolution of the Wigner function for a system of coupled kicked rotators, and the mean-field approximation is introduced. In the third section the mean-field equation is replaced by an equivalent system of a quantum kicked rotator coupled with a classical kicked rotator representing the environment. And numerical simulation of this system is examined. The idea of spontaneous decoherence is discussed based on the numerical calculation in the last section.

2 Large N limit and the statistical mean-field approximation

Let us examine the scenario more explicitly by taking the coupled N-kicked rotator model [8] as an example. The Hamiltonian is assumed to be

$$H = \sum_{i=1}^{N} \{\frac{1}{2} t_{ii} p_i^2 + \Delta(t) \lambda_i \cos\theta_i\} + \frac{1}{2} \sum_{i \neq j}^{N} t_{ij} p_i p_j, \tag{6}$$

where

$$\Delta(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$$

is the periodical kick. In this model couplings of N kicked rotators enter through off-diagonal kinetic terms. Couplings through the kick term [9] may be converted to this form through a linear transformation of angle variables. The classical equation of motion is known as the *standard map* which is the mapping between canonical coordinates just after kicks at times t and $t+1$,

$$\left\{ \begin{array}{lcl} \theta_i^{(t+1)} & = & \theta_i^{(t)} + \sum_j t_{ij} p_j^{(t)} \\ p_i^{(t+1)} & = & p_i^{(t)} + \lambda_i \sin\theta_i^{(t+1)}. \end{array} \right. \qquad (t : integer) \tag{7}$$

We shall choose the magnitudes of matrix elements of the kinetic term as

$$\begin{aligned} t_{ii} &\sim \mathcal{O}(1), \\ t_{ij} &\sim \mathcal{O}(\tfrac{1}{N^\nu}) \text{ for i} \neq \text{j}, \end{aligned} \tag{8}$$

where the exponent ν will be determined later so that the system is non-trivial and converging in the large N limit.

Evolution of the Wigner function is obtained by solving the Schrödinger equation. The relation between Wigner functions just after kicks at times t and $t+1$ is called the quantum mapping, and it can be written as

$$f^{(t+1)}(\{\theta_i\},\{p_i\}) = UVf^{(t)}(\{\theta_i\},\{p_i\}) \tag{9}$$

with the mapping operators U and V defined by

$$U = \exp(-\sum_i \lambda_i \sin\theta_i \frac{D}{Dp_i}), \tag{10}$$

and

$$V = \exp(-\sum_{ij} t_{ij} p_i \frac{\partial}{\partial\theta_j}). \tag{11}$$

Here, the operator $\frac{D}{Dp}$ appeared in U,

$$\frac{D}{Dp} \equiv \frac{1}{\hbar}(e^{\frac{\hbar}{2}\frac{\partial}{\partial p}} - e^{-\frac{\hbar}{2}\frac{\partial}{\partial p}}), \tag{12}$$

is a difference operator, and quantum effects in the evolution equation enter only through this operator. Since it approaches to the differential operator in the classical limit as

$$\begin{aligned} \frac{D}{Dp}f(p) &= \frac{1}{\hbar}[f(p+\frac{\hbar}{2}) - f(p-\frac{\hbar}{2})] \\ &\sim \frac{\partial}{\partial p}f(p), (\hbar \to 0) \end{aligned} \tag{13}$$

the quantum mapping reduces to the well-known standard map of classical distribution function,

$$f^{(t+1)}(\{\theta_i\},\{p_i\}) \approx f^{(t)}(\{\theta_i - \sum_j t_{ij}(p_j - \lambda_j \sin\theta_j)\},\{p_i - \lambda_i \sin\theta_i\}) \tag{14}$$

in the $\hbar \to 0$ limit.

Although the difference between classical and quantum evolutions looks very small from eqs.(10) and(13) at a glance, the consequence of actions of the operator U on the exponential function may be instructive. In quantum mechanical case it acts as

$$e^{(-a\frac{D}{Dp})}e^{ikp} = e^{ik\{p-a\varphi(k)\}}, \tag{15}$$

with

$$\varphi(k) = \frac{2}{\hbar k}\sin\frac{\hbar k}{2}, \tag{16}$$

while in the classical limit it reduces to the standard shift operator along the momentum axis,

$$e^{-a\frac{\partial}{\partial p}}e^{ikp} = e^{ik(p-a)}, \tag{17}$$

i.e. the operator $\frac{D}{Dp}$ behaves as the usual shift operator only for small wave number components. For large wave numbers shifts are suppressed by the factor $1/k$ and direction of the shift alternates. This implies that rapid variations of the Wigner function along the p-axis are suppressed, and consequently chaotic folding of the distribution function terminates at fine scale of the order $\hbar$.

Now, let us consider the large N limit where we expect that the mean-field approximation works well. Let us suppose at a certain time t the Wigner function is written in a product form,

$$f^{(t)}(\{\theta_i\},\{p_i\}) = \prod_{i=1}^{N} f_i^{(t)}(\theta_i, p_i). \tag{18}$$

Integrating the evolution equation(9) over all phase volume except the i-th canonical coordinates, we obtain the equation for the reduced Wigner function of the i-th subsystem:

$$\begin{aligned} f_i^{(t+1)}(\theta_i, p_i) &= \exp(-\lambda_i \sin\theta_i \frac{D}{Dp_i})\exp(-t_{ii}p_i\frac{\partial}{\partial\theta_i}) \\ &\times \int \prod_{j(\neq i)} d\theta_j dp_j \exp(-\sum_{j(\neq i)} t_{ij}p_j\frac{\partial}{\partial\theta_i}) f^{(t)}(\{\theta_j\},\{p_j\}), \end{aligned} \tag{19}$$

where the reduced Wigner function has been defined at $t+1$ as

$$f_i^{(t+1)}(\theta_i, p_i) = \int \prod_{j(\neq i)} d\theta_j dp_j f^{(t+1)}(\{\theta_j\},\{p_j\}), \tag{20}$$

which will turn out to be the solution of the equation which we may call the *statistical* mean-field equation evolved from $f_i^{(t)}(\theta_i, p_i)$ of eq.(18).

Expanding the exponential function in the integrand of the r.h.s. of eq.(19) we obtain

$$\Bigg[1-\sum_{j(\neq i)} t_{ij}\langle p_j\rangle_j^{(t)}\frac{\partial}{\partial\theta_i} + \frac{1}{2}\sum_{j,k(j\neq k,\neq i)} t_{ij}t_{ik}\langle p_j\rangle_j^{(t)}\langle p_k\rangle_k^{(t)}\frac{\partial^2}{\partial\theta_i^2}+\frac{1}{2}\sum_{j(\neq i)} t_{ij}^2\langle p_j^2\rangle_j^{(t)}\frac{\partial^2}{\partial\theta_i^2}\Bigg] f_i^{(t)}(\theta_i,p_i),$$

for the first and second order terms, where we have written

$$\langle p_j^n\rangle_j^{(t)} = \int d\theta_j dp_j p_j^n f_j^{(t)}(\theta_j,p_j). \tag{21}$$

Writing back this expression to the exponential form we have

$$\exp(-\sum_{j(\neq i)} t_{ij}\langle p_j\rangle_j^{(t)}\frac{\partial}{\partial\theta_i}+\sum_{j(\neq i)} t_{ij}^2\sigma_j^{(t)}\frac{\partial^2}{\partial\theta_i^2})f_i^{(t)},$$

for the first two terms of the cumulant expansion. Here, $\sigma_j^{(t)}$ is the momentum variance of the j-th sub-system,

$$\sigma_j^{(t)} = \langle p_j^2\rangle_j^{(t)} - \{\langle p_j\rangle_j^{(t)}\}^2. \tag{22}$$

We have chosen that the coupling strength $t_{ij}(i\neq j)$ to be $\mathcal{O}(N^{-\nu})$, and thus the n-th cumulant (*i.e.* the term involving $\frac{\partial^n}{\partial\theta_i^n}$) is of the order of $N^{1-n\nu}$. The parameter ν classifies nature of the system into the following cases in the $N\to\infty$ limit:

i) $\nu>1$, any cumulant converges to 0 and the system becomes a set of independent kicked rotators.

ii) $\nu=1$, only the first cumulant $\sum_{j(\neq i)} t_{ij}\langle p_j\rangle_j^{(t)}$ survives and the *ordinary* mean-field Hamiltonian,

$$\bar{h}_i = h_i + \sum_{i\neq j} p_i t_{ij}\langle p_j\rangle_j^{(t)}, \tag{23}$$

drive the reduced Wigner function $f_i^{(t)}$.

iii) $\frac{1}{2}<\nu<1$, the ordinary mean-field theory is valid assuming $\sum_{j(\neq i)} t_{ij}\langle p_j\rangle_j^{(t)}$ remains finite. By transforming the system in a rotating frame with an appropriate momentum $\bar{p}$, the fluctuation $\sum_{j(\neq i)} t_{ij}(\langle p_j\rangle_j^{(t)}-\bar{p})$ can be kept finite.

iv) $\nu=\frac{1}{2}$, the statistical mean-field theory is valid providing $\sum_{j(\neq i)} t_{ij}(\langle p_j\rangle_j^{(t)}-\bar{p})$ kept finite.

v) $\nu < \frac{1}{2}$, the second cumulant diverges and the system becomes unphysical. In this case it does not make any sense to consider the reduced Wigner function.

The statistical mean-field equation is given by

$$f_i^{(t+1)}(\theta_i, p_i) = \exp(-\lambda_i \sin\theta_i \frac{D}{Dp_i}) \exp(-t_{ii} p_i \frac{\partial}{\partial\theta_i})$$
$$\exp(-\sum_{j(\neq i)} t_{ij} \langle p_j \rangle_j^{(t)} \frac{\partial}{\partial\theta_i} + \sum_{j(\neq i)} t_{ij}^2 \sigma_j^{(t)} \frac{\partial^2}{\partial\theta_i^2}) f_i^{(t)}(\theta_i, p_i). \tag{24}$$

Since evolution operator of the i-th subsystem involves only the i-th canonical coordinates product form of the Wigner function will remains at $t+1$:

$$f^{(t+1)}(\{\theta_i\}, \{p_i\}) = \prod_{i=1}^{N} f_i^{(t+1)}(\theta_i, p_i). \tag{25}$$

The proof that the product eq.(25) is indeed the solution of the evolution equation (9) is given in ref. [7].

For studying asymptotic behavior of the momentum distribution it is more convenient to consider the Fourier transform of the reduced Wigner function in the statistical mean-field approximation,

$$\tilde{f}_i^{(t)}(m_i, k_i),$$

where m_i and k_i are the Fourier parameters corresponding to the variables θ_i and p_i, respectively. The momentum distribution function for the i-th sub-system is then readily expressed as

$$f_i^{(t)}(p) = \int_0^{2\pi} \frac{dk}{2\pi\hbar} e^{-ikp/\hbar} \tilde{f}_i^{(t)}(0, k). \tag{26}$$

The evolution equation in this representation is written as

$$\tilde{f}_i^{(t+1)}(m_i, k_i) = \sum_{m_i'} J_{m_i - m_i'}(\frac{2\lambda_i}{\hbar} \sin\frac{\hbar k_i}{2}) e^{-i m_i' g_i^{(t)} - \frac{1}{2} m_i'^2 \Sigma_i^{(t)}} \tilde{f}_i^{(t)}(m_i', k_i - m_i'), \tag{27}$$

where $J_m(k)$ is the Bessel function of the order m, and we have defined

$$g_i^{(t)} = \sum_{j(\neq i)} t_{ij} \langle p_j \rangle_j^{(t)}, \tag{28}$$

$$\Sigma_i^{(t)} = \sum_{j(\neq i)} t_{ij}^2 \sigma_j^{(t)}. \tag{29}$$

For the asymptotic behavior of $\Sigma_i^{(t)}$, there will exist at least two cases depending on the nature of environment:

$$\begin{aligned}\sigma_i^{(t)} &\rightarrow const.\text{(regular)} \\ &\rightarrow d_i t. \quad \text{(chaotic)}\end{aligned} \tag{30}$$

When the environment is chaotic the sum over m_i in eq.(27) will be dominated asymptotically by the $m_i = 0$ components. The asymptotic form of the evolution equation then becomes

$$\tilde{f}_i^{(t+1)}(m_i, k_i) \sim J_{m_i}(\frac{2\lambda_i}{\hbar} \sin \frac{\hbar k_i}{2}) \tilde{f}_i^{(t)}(0, k_i), \tag{31}$$

whose solution is now trivial. Especially the $m_i = 0$ components are given as

$$\tilde{f}_i^{(t)}(0, k_i) \sim \{J_0(\frac{2\lambda_i}{\hbar} \sin \frac{\hbar k_i}{2})\}^t \tilde{f}_i^{(0)}(0, k_i). \tag{32}$$

Inserting this solution into eq.(26) together with help of the cental limit theorem we obtain the asymptotic form of the momentum distribution function as

$$f_i^{(t)}(p) \sim [\frac{1}{\pi t \lambda_i^2}]^{\frac{1}{2}} e^{-\frac{p^2}{t\lambda_i^2}}, \tag{33}$$

which is the same function as the classical distribution of the chaotic limit. The $m = 0$ dominance suggests that when the environment become chaotic the system starts behaving classically due to the phase decoherence.

3 An equivalent model and numerical results

We have seen that the N-coupled kicked rotator will fall into chaos in the large N limit in cases when the statistical mean-field approximation works well. Under those circumstances it becomes possible to study, instead of treating whole system at once, just by picking up any one of the rotators as an object system which we simply call the system(S) under influence of other rotators as the environment(E). Interaction between S and E enters as an external field with strength proportional to the state averaged momentum in our model. Although the mean-field approximation makes it possible to perform the large N calculation as many as $N \approx 100 - 1000$ by using large scale parallel processors, we leave such huge simulations as the future project. Instead, we assume the mean-field strength (28) is replaced by a classical system with appropriate initial ensemble. We choose the ensemble so that it become equivalent to the statistical mean-field theory.

Now, let us consider a quantum kicked rotator(S) with the Hamiltonian,

$$H_S = \frac{1}{2}p^2 + \Delta(t)\lambda_S \cos\theta + g^{(t)}p \tag{34}$$

under the influence of classical environment(E) through $g^{(t)}$. This system evolves as

$$f_S^{(t+1)}(\theta, p) = \exp(-\lambda_S \sin\theta \frac{D}{Dp}) \exp(-p\frac{\partial}{\partial\theta}) \exp(-g^{(t)}\frac{\partial}{\partial\theta}) f_S^{(t)}(\theta, p), \tag{35}$$

or by the Fourier transform it reads

$$\tilde{f}_S^{(t+1)}(m, k) = \sum_{m'} J_{m-m'}(\frac{2\lambda_S}{\hbar}\sin\frac{\hbar k}{2}) e^{-im'g^{(t)}} \tilde{f}_S^{(t)}(m', k-m'). \tag{36}$$

After taking average over the initial ensemble the evolution equation is written as

$$E[\tilde{f}_S^{(t+1)}(m, k)] = \sum_{m'} J_{m-m'}(\frac{2\lambda_S}{\hbar}\sin\frac{\hbar k}{2}) E[e^{-im'g^{(t)}} \tilde{f}_S^{(t)}(m', k-m')], \tag{37}$$

where we write the ensemble averaged quantities by $E[\;\;]$. We assume the ensemble of mean-field couplings $\{g^{(t)}\}$ to be Gaussian with the correlation,

$$E[g^{(t)}g^{(t')}] - E[g^{(t)}]E[g^{(t')}] = \delta_{tt'}\sigma^{(t)}. \tag{38}$$

In this case the correlation between $g^{(t)}$ and $\tilde{f}^{(t)}$ vanishes, and we have

$$E[\tilde{f}_S^{(t+1)}(m, k)] = \sum_{m'} J_{m-m'}(\frac{2\lambda_S}{\hbar}\sin\frac{\hbar}{2}k) e^{-im'E[g^{(t)}]-\frac{1}{2}m'^2\sigma^{(t)}} E[\tilde{f}_S^{(t)}(m', k-m')]. \tag{39}$$

This equation is equivalent to the statistical mean-field equation (27).

Our next task is to give $g^{(t)}$ explicitly in order to solve the evolution eq.(35). One of the possible choice of $g^{(t)}$ which possesses both properties eq.(30) and (38) is the momentum generated by the classical kicked rotator,

$$\left\{ \begin{array}{lcl} \phi^{(t+1)} & = & \phi^{(t)} + g^{(t)} \\ g^{(t+1)} & = & g^{(t)} + \lambda_E \sin\phi^{(t+1)} \end{array} \right. \tag{40}$$

with a set of initial values $(\phi^{(0)}, g^{(0)})$ as the ensemble. Iterating eq.(35) together with eq.(40) the behavior of S will show the quantum-classical transition depending on the environment.

Before examining results of the numerical calculation let us remark the consistency of the choice of $g^{(t)}$. The momentum distribution of E in the mean-field approximation is given by

$$f_E^{(t)}(p_E) = \int \prod_{j(\neq S)} dp_j \delta(p_E - \sum_{j(\neq S)} p_j) \prod_{j(\neq S)} f_j^{(t)}(p_j). \tag{41}$$

If each rotator is in a chaotic state its momentum distribution is known to be

$$f_j^{(t)}(p) \propto e^{-\frac{p^2}{td_j}} \tag{42}$$

asymptotically. In this case $f_E^{(t)}(p_E)$ also has the same form with the diffusion constant being

$$d_E = \sum_{j(\neq S)} t_{Sj}^2 d_j, \tag{43}$$

which is $\mathcal{O}(1)$ for our choice of $t_{Sj} \approx \mathcal{O}(\frac{1}{N^{1/2}})$.

We shall now examine numerical results which support the scenario described in the last section. We solve the Schrödinger equation with the Hamiltonian (34) with $g^{(t)}$ generated from the standard map (40). At each time step we measure the momentum fluctuation of S,

$$E[\langle p^2 \rangle^{(t)}],$$

averaged over the initial ensemble which consists typically 20 members for each set of parameters λ_S, λ_E and μ. As in the classical rotator chaos is measured by the diffusion constant defined by the rate of momentum fluctuation.

In order to see the effect of chaotic environment the system is maintained in chaos by choosing $\lambda_S = 6.0$, and varying the environment λ_E (Fig.1). The coupling strength is chosen to be $\mu = 0.01$. As we have expected the energy diffusion starts precisely when the environment become chaotic, *i.e.* $\lambda_E \approx 1.0$. To make sure that the increase of kinetic energy is not due to the direct penetration of energy from E via coupling, we made the similar calculation by keeping the system in the regular regime, $\lambda_S = 0.8$. No energy diffusion has been observed in this case.

Choice of the coupling strength $\mu = 0.01$ comes from the requirement that the property of S should not be altered significantly by the environment. Weakness of the coupling effect is clear if we compare diffusion constants of an isolated classical rotator which is 18.0 for $\lambda = 6.0$. No significant difference is observed when the coupling strength is less than 0.01. Although the effect of environment is small on the classical system, it has the essential action on the quantum system. It is seen in Fig.2 where S and E are kept in chaos ($\lambda_S = \lambda_E = 6.0$) and the coupling strength is varied. Above the lower critical coupling $\mu \approx 0.001$ the environment effect appears in the diffusion constant of S.

Next, we observe the transition of S from the regular motion to chaos under the influence of the chaotic environment with keeping $\lambda_E = 6.0$ (Fig.3). At the classical

critical strength $\lambda = 1.0$ the quantum system persists to be diffusion-less. It is only after $\lambda \approx 3.0$ the system shows diffusion. Change of the evolution from quantum to classical by the effect of E can also be seen in the momentum distribution. Figs.4 (a) and (b) correspond to the regular motion and the chaotic regime, respectively. In the regular case the momentum localization persists during the simulation time up to $t < 300$, while in the chaotic case the momentum distribution changes following the classical diffusion law described by eq.(33).

4 Spontaneous decoherence -A conclusion-

It is well-known that a single quantum kicked rotator shows only limited energy diffusion. The limited diffusion is considered to be generic in quantized systems with a few degrees of freedom whose motion show chaos in the classical evolution. We have searched the source of classical diffusion in systems with infinite number of degrees of freedom. These system can be treated in the statistical mean-field approximation, where we can pick any one of the sub-system as an object system and the rest as the environment. Energy diffusion has been observed in the quantum system when the environment is in classical chaos. Momentum distribution of the quantum system is also same as one which we normally observe in a classical chaotic system. Since each building block of the environment is the same type of quantum system, the transition from the quantum behavior to the classical behavior seen in the diffusion should occur spontaneously.

One may wonder then where is the seed of chaos in linear dynamical system like quantum mechanical system. It is certainly very difficult to point out the cause of chaos unlike classical dynamical system where the existence of homoclinic point means chaos. Although we have no rigorous proof for the existence of chaos in quantum systems in the limit of infinite number of degrees of freedom, the exactness of the certain class of mean-field approximation in the large N limit suggests that the non-linearity is secretly inherited in quantum system with infinite number of degrees of freedom. It is the non-linear mean-field equation which possesses the source of chaos within itself.

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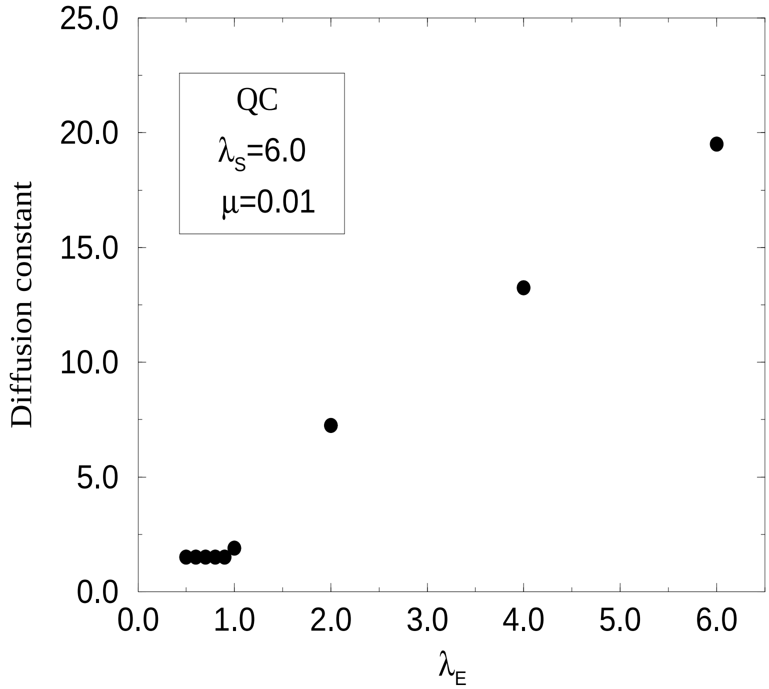

Figure 1: Diffusion constants of S fixing the kick strength in the chaotic regime($\lambda_S = 6.0$) and varying λ_E to change the environment from regular to chaos.

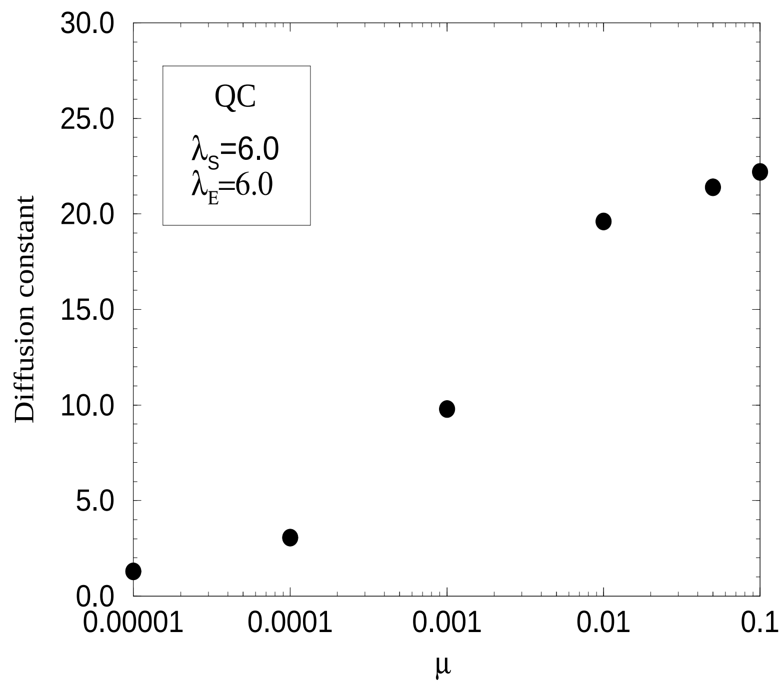

Figure 2: Diffusion constants of S fixing the kick strength of S and E both in the chaotic regime($\lambda_S = \lambda_E = 6.0$), and varying the coupling μ.

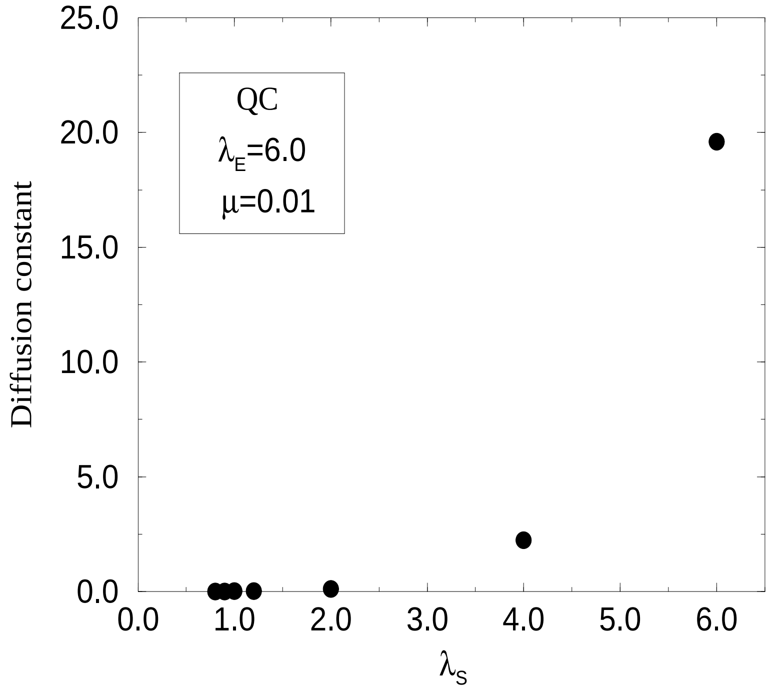

Figure 3: Diffusion constants of S fixing the kick strength of E in the chaotic regime($\lambda_E = 6.0$), and varying λ_S.

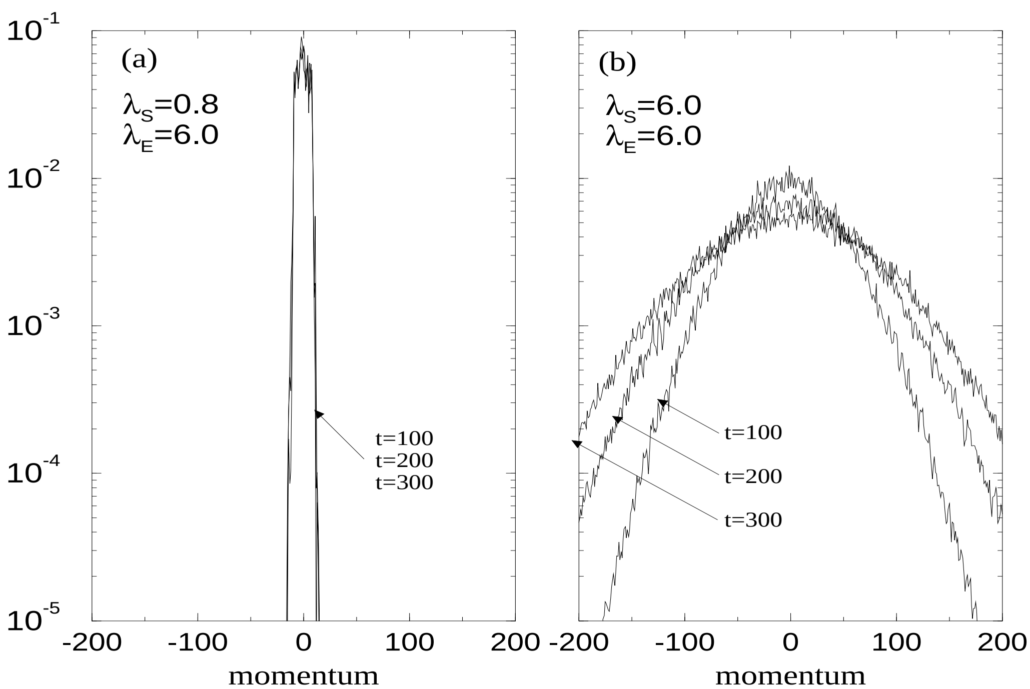

Figure 4: Time evolutions of the momentum distributions of S setting the kick strength of E in the chaotic regime($\lambda_E = 6.0$) and keeping the system in (a) the regular motion ($\lambda_S = 0.8$), or (b) the chaotic motion ($\lambda_S = 6.0$). Both of (a) and (b) are given by 100 ensembles average.